



REVIEW

on the competition for the position of Associate Professor in the field of higher education: 4. Natural Sciences, Mathematics and Informatics, professional direction: 4.5. Mathematics, scientific specialty "Geometry and Topology", announced in SG no. 97 of 21.11.2023
by Stefan Ivanov, appointed according to Order No. RD - 16-43/19.01..2024 of the Rector of the Agricultural University - Plovdiv as a member of the scientific jury

Reviewer: The review was prepared by Prof. Dr. Sci. Stefan Petrov Ivanov - FMI at SU and IMI-BAN, professional direction 4.5 Mathematics, scientific specialty "Geometry and Topology" (Differential Geometry), in my capacity as a member of the scientific jury for the competition according to Order No. RD -16-43 /19.01..2024 of the Rector of the Agricultural University - Plovdiv.

Only one candidate submitted documents for participation in the announced competition:

Ch. Assistant Professor Dimitar Rumenov Razpopov, PhD, AU Plovdiv.

2. General description of the presented materials.

The documents submitted by the candidate in the competition correspond to the requirements of the ZRASRB, PPZRASRB and the Regulations for Academic Development in AU-Plovdiv (RADAU) include:

- Application to the Rector of AU-Plovdiv for admission to the competition with a list of documents.
- Curriculum vitae in European format (CV).
- Copies of diplomas with attachments for completed bachelor's and master's degrees (notarized).
- Copy of the diploma for the acquired educational and scientific degree "doctor" in the scientific specialty of the announced academic position (notarized).
- Certificate of internship in the relevant specialty, reference for the academic

positions held to date, service notes for conducting lectures and exercises as a part-time teacher.

- Order of the rector (director) of a higher school or scientific organization to confirm the selection for the occupation of the academic position "chair assistant".

- Copy of the advertisement in the state gazette.

- General list of publications.

- List of post-doctoral publications and copies of them.

- Summaries of submitted publications.

- List of citations.

- Information on impact factor.

- Reference of contributions.

- Reference to teaching experience, teaching employment, participation in projects and published textbooks, etc.

- Reference-declaration on the fulfilment of the minimum national scientific-metric requirements under Article 2b, paragraphs 2 and 3 of the ZRASRB.

- Declaration of originality and authenticity of the attached documents.

- Other documents on academic research and administrative activities.

- Information card for the NACID.

To participate in the competition, the candidate ch. Assistant Professor Dimitar Rumenov Razpopov, Ph.D., presented a list of 12 scientific publications in Bulgarian and international journals, 1 book based on his doctoral dissertation and 2 university textbooks, including 9 publications on the topic of the competition, of which 2 with an impact factor, 1 with SJR in the well-known international journal Journal of Geometry, 5 in peer-reviewed editions and 1 in a non-refereed edition. A list of citations and other documents (in the form of memos and certificates from an employer, project manager, funding organization or project contractor, references and testimonials, awards and other relevant evidence) supporting the applicant's achievements is presented.

For the preparation of the review, 9 issues are subject to analysis.

I will note that the Journal of Geometry does not have an impact factor for 2020, but it has a JCI=0.63, which does not significantly change the points in the NACID map.

3. Brief biographical data of the applicant:

Ch. assistant professor Dimitar Rumenov Razpopov, PhD, was born in 1973. He received his secondary special education at TMT "Kalinin" in Plovdiv in 1992. During the period 1992-1999 he is a student at FMI of "Paisiy Hilendarski" PU, majoring in "Mathematics and Informatics" and received a master's degree. From 2000 to 2004 was a computer science teacher in various secondary schools in the city of Plovdiv. In 2004, he became an assistant, in 2006 - a senior assistant, and in 2008 - a chief assistant at AU - Plovdiv, where he has been working until now. From 2019-2023 he is also the main assistant at the University of Food Technologies (UHT) - Plovdiv. At AU and OHT - Plovdiv, he gave lectures and gave exercises to students in higher mathematics, applied mathematics, statistics, game theory, financial mathematics and business logistics. During the period 2015-2018, he was a doctoral student of independent training at FMI at PU "Paisiy Hilendarski", PhD program "Geometry and Topology", where he defended his doctoral dissertation in professional direction 4.5. Mathematics in 2019.

He has 11 participations in scientific conferences in the country and abroad and 19 scientific articles.

4. General characteristics of the scientific works and achievements of the candidate

Ch. Assistant Professor Dimitar Razpopov works in the field of the differential geometry of Riemannian manifolds equipped with circulant structures, one of the new modern and difficult to study directions in differential geometry in recent years. A major problem in studying the differential geometry of a smooth manifold with structures is the construction of non-trivial examples and their study in order to discover and confirm the corresponding geometric theory, which is a highly non-trivial problem. For example, the theory of surfaces in three-dimensional Euclidean space and, accordingly, the differential geometry of Riemannian manifolds started after Riemann's construction of the metric on the sphere, analogously is the theory of Kähler manifolds, etc.

Dr. Dimitar Razpopov's research is devoted to these topical and non-trivial problems in the case of a Riemannian manifold with circulant structures. These manifolds are a special class of n -dimensional Riemannian manifolds with Riemannian metric g and

a circulant tensor field q of type $(1,1)$ whose n th degree is equal to the identity, $q^n = 1$ such that the circulant structure q is an isometry, $g(QX, QY) = g(X, Y)$. Circularity means that in a local coordinate system the matrices of the structures are circulant matrices, i.e. each subsequent row is obtained by cyclic permutation of the previous one. Circulant matrices and skew-circulant matrices, also known as Töplitz matrices, are considered and have been studied in detail by R. Gray and P. Davies. It is known that non-singular right circulant (skew-circulant) matrices form a multiplicative group and have important applications in geometry, coding, graph theory, vibration analysis.

With this, I believe that the topic in which the candidate works is sufficiently relevant on a global scale.

For participation in the competition the candidate Dimitar Razpopov, Ph.D. has submitted 12 scientific publications, 9 of which are on the subject of the competition.

There are presented 19 citations of the candidate's works.

He has participated in 4 scientific projects.

All scientific works presented by the candidate do not repeat those from previous procedures for obtaining a scientific title and academic position.

There is no evidence of plagiarism in the scientific works submitted to the competition.

Obviously, the submitted scientific works significantly exceed the minimum national requirements (according to Article 2b, paragraphs 2 and 3 of the Law on Research and Development) and, respectively, the additional requirements of AU-Plovdiv for the academic position of Associate Professor in the scientific field and professional field of the competition.

5. Characteristics and evaluation of the candidate's teaching activity

The candidate has a rich and fruitful teaching and pedagogical activity at the Agricultural University, Plovdiv and the University of Food Technologies (UXT) - Plovdiv. At the AU and UHT - Plovdiv he has lectured and taught students in higher mathematics, applied mathematics, statistics, game theory, financial mathematics and economic logistics. His rich teaching activity is supported by 2 textbooks for students.

6. Analysis of the applicant's scientific and scientific-applied achievements contained in the materials for participation in the competition

Ch. Assistant Professor Dimitar Razpopov works in the field of the differential geometry of Riemannian manifolds equipped with circulant structures, and his investigations are important for the development of the differential geometry of these spaces and provide the basis for further development of these remarkable and difficult to study manifolds.

In the work [rec. 4] the geometry of three-dimensional and four-dimensional Riemannian manifolds with circulant structure q and Riemannian metric g , $q^n = 1$, $n=3,4$, respectively is studied. Curvature invariants are calculated and conditions for belonging to the main curvature classes of three-dimensional and four-dimensional Riemannian manifolds with the corresponding circulant structure are determined.

In dimension 3, the Riemannian metric and the circulant structure define an indefinite metric (adjoint metric). For an isotropic vector field with respect to the adjointed metric, the corresponding force acting on isotropic curves is considered and the corresponding "physical work" defined by the corresponding integral along the curve is studied [rec. 3]. Analogous questions about "physical work" in dimension 4 are discussed in [rec. 2], and the obtained results are non-trivial and differ from the three-dimensional case.

In dimension 3, a circulant metric with a circulant structure is also considered, curvature properties are investigated, and a non-trivial example is constructed [ref. 5].

The geometry of 4-dimensional Riemannian manifolds with Riemannian metric g and right skew-circulant structure S , with fourth degree equal to -1 , $S^4 = -1$, acting on the metric as an isometry, $g(S., S.) = g(., .)$ was studied in works [1], [2], [3] and [rec. 1]. In this case, the Riemannian metric is by necessity right skew circulant.

In my opinion, the main scientific contribution of the candidate is contained in [1]. In the work [1], the basic case when the skew-circulant structure is parallel to the Levi-Civita connection is considered, necessary and sufficient conditions are given in terms of the skew-circulant metric and its first partial derivatives (Theorem 6.2), curvature identities are defined in the circulant basis, the components of the Ricci tensor are calculated and

it is proved that the manifold is almost Einstein, i.e. the Ricci tensor is defined as a linear combination of the metric and its adjoint metric (Theorem 3.4). As a consequence, a formula for the curvature tensor in terms of the metric and its adjoint metric is found (Theorem 3.6). It is proved that the almost complex structure defined by the formula $J=S^2$ is Kählerian if and only if the skew-circulant structure S is parallel to the Levi-Civita connectivity (Theorem 6.3). A nontrivial example of a parallel skew-circulant structure on a 4-dimensional real Lie group is constructed.

In the work [rec. 1] it was established that the almost complex structure defined by the formula $J=S^2$ is always locally conformal Kähler and, in particular, it is integrable. It is worth noting that the properties of these manifolds with respect to the indefinite neutral adjoint metric are discussed in [3], defining and investigating indefinite hyperspheres and special infinite circles, and in [2] Green's theorem for circles in special two-dimensional planes in the tangent space of a point of the manifold is interpreted.

In conclusion, I believe that the developments in the candidate's works are important for the development of (right skew) circulant differential geometry and provide the basis for further development of these remarkable and difficult to be investigated manifolds having the potential for significant applications in differential geometry and mathematical physics .

7. I have essentially no critical notes and recommendations.

8. Conclusion.

The candidate in the competition has submitted a sufficient number of scientific works published after the materials used in the defense of the scientific degree "doctor". Ch. assistant professor Dr. Dimitar Razpopov works in the modern field of differential geometry and his results in recent years could be significantly used in modern mathematics and mathematical physics. The candidate's works contain original scientific and applied contributions that have received international recognition, as a representative part of them have been published in well-known scientific journals and scientific collections issued by international academic publishing houses.

In my opinion, the scientific and teaching qualifications of Ch. Ass. Dimitar Razpopov is undoubted.

9. Conclusion on the application

Having read the materials and scientific works submitted in the competition and based on the analysis of their significance and the scientific and applied contributions contained therein, I **confirm** that the scientific achievements meet the requirements of the Law on Research and Development, the Regulations for its application and the relevant Regulations of the Agricultural University of Plovdiv for the candidate to hold the academic position of Associate Professor in the scientific field and professional field of the competition.

In particular, the candidate satisfies the minimum national requirements in the professional field of the competition and no plagiarism has been found in the scientific works submitted for the competition.

I give my **positive assessment** to the application.

II. GENERAL CONCLUSION

On the basis of the above, I **recommend** the scientific jury to propose to the competent body for the selection of the Agricultural University - Plovdiv to elect ch. asst. Dr. Dimitar Rumenov Razpopov to occupy the academic position of Associate Professor in the professional field 4.5. Mathematics (Geometry and Topology).

16.02.2024r.

REVIEWER:.....

(Prof. Dr. Sci. Stefan Ivanov)